CIRCULAR MODEL (CM)

The Circular Model was introduced in year 2016. It was a part of the Doctor of Philosophy (PhD in Statistics) thesis of W. G. Samanthi Konarasinghe, University of Peradeniya, Sri Lanka. Development of the CM was based on theory of Uniform Circular motion, Fourier Transformation and Least Square Regression Analysis. Fourier transformation (FT) can be used to transform a real valued function f(x) into series of trigonometric functions.

FT has two versions; discrete transformation and continuous transformation. The discrete version of Fourier transformation is; $f(x) = \sum_{-\infty}^{\infty} a_n e^{-k\theta}$

According to De Moivre's theorem; $e^{-k\theta} = \cos k\theta \ + i \sin k\theta$

Where, i is a complex number. Therefore *f(x)* can be written as:

$$f_x = \sum_{k=1}^{n} a_k \cos k\theta + b_k \sin k\theta \tag{1}$$

Whare a_k and b_k are amplitudes, k is the harmonic of oscillation.

A particle *P,* which is moving in a horizontal circle of centre O and radius a is given in Figure 1, ω is the angular speed of the particle;

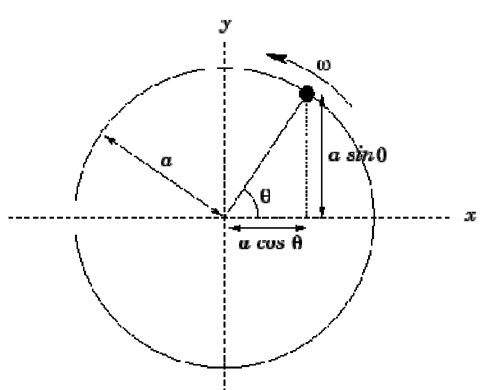

Figure 1 - Motion of a particle in a horizontal circle

Angular speed is defined as the rate of change of the angle with respect to time. Then;

$$\omega = \frac{d\theta}{dt}$$

$$\int_0^\theta d\theta = \int_0^{\mathrm{t}} \omega \, \mathrm{dt}$$

Hence, $\theta = \omega t$ (2)

Substitute (2) in (1); $f_x = \sum_{k=1}^{n} a_k \cos k\omega t + b_k \sin k\omega t$ (3)

At one complete circle θ=2π radians. Therefore, the time taken for one complete circle (*T*) is given by: $T = 2\pi / \omega$ (4)

In circular motion, the time taken for one complete circle is known as the period of oscillation. In other words, the period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with *f* peaks in *N* observations, its period of oscillation can be given as;

$$T = \frac{\textit{total}\text{ number of periods}}{\text{total number of peaks}} = \frac{N}{f} \quad (5)$$

Equating (4) and (5); $\frac{2\pi}{\omega} = \frac{N}{f}$

Then, $\omega = 2\pi \frac{f}{N}$

However, (1) it is a deterministic model, does not capture the randomness in real life. Therefore (1) is modified as follows;

$$Y_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \qquad (6)$$

The model (6) was named as “Circular Model”.

Model Assumptions:

Y_t is a continuous random variable

$t \geq 0$,

$k \in Z^+$

Series, sin $k\omega t$ cos $k\omega t$ are independent

ε is Normally distributed d with 0 mean and constant variance

ε is Independent

The CM can be applied to model a regular or irregular wave with no trends.